

# Design of Spacecraft Flywheel Fault Detection System Based on Attitude Fault Tolerant Control

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**Abstract:** To solve the problems of actuator failure and external interference of rigid spacecraft with four reaction flywheels, an attitude fault-tolerant control assignment algorithm is proposed. In this method, a sliding mode observer is designed to realize the precise reconstruction of actuator faults and external disturbances in a finite time. In particular, the Lyapunov stability theory is applied to prove that the designed controller can achieve the global asymptotically stable control of the closed-loop attitude in a finite time, and the control strategy can achieve the robustness to the reaction flywheel faults and external disturbances. In addition, the method of base order optimal control assignment with less computation is used to realize the assignment of the desired control torque to the command control torque of the four reaction flywheel. Finally, the numerical simulation of a certain type of spacecraft and various reaction flywheel faults is carried out. The simulation results show that the designed flywheel fault reconstruction and attitude fault-tolerant control method can complete the fault reconstruction and control allocation online, timely and accurately.

## 1. Introduction

A fault reconstruction and attitude fault-tolerant controller design method with redundant reaction flywheels is proposed for the control problem of spacecraft in orbit affected by external disturbance torque and actuator failure. The designed sliding mode observer and control law can guarantee the accurate reconstruction and robustness of faults (idling, stalling, increasing friction torque and flywheel efficiency) in real time in a limited time. In addition, this paper takes the real-time optimal allocation of redundant flywheels as the research goal, and uses the basic sequence optimal algorithm with simple algorithm, high efficiency and suitable for the application of spacecraft in orbit to achieve the distribution of the desired control torque. At the same time, this method solves the problems in the existing control allocation, such as the complexity of calculation, the weakening of the physical meaning of variables, and the difficulty of adaptive adjustment. In the design process of the control allocation algorithm, the problems of limited output torque amplitude and speed of the actuator are considered, which makes the method more meaningful for engineering implementation. Finally, the method of fault reconstruction and control allocation is applied to the attitude control task of a spacecraft, which can realize the precise reconstruction of reaction flywheel fault, achieve the optimal allocation of desired control and achieve the control goal.

## 2. Mathematical model of attitude of spacecraft

This paper adopts the Euler angle formula is used to describe the attitude of the spacecraft, and the mathematical model of its attitude can be expressed as [8] below:

$$\dot{\omega} = R(\Theta)\dot{\Theta} - \omega_c(\Theta) \quad (1)$$

$$I_1 \dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 = u_1 + d_1 \quad (2)$$

$$I_2 \dot{\omega}_2 - (I_3 - I_1) \omega_1 \omega_3 = u_2 + d_2 \quad (3)$$

$$I_3 \dot{\omega}_3 - (I_1 - I_2) \omega_1 \omega_2 = u_3 + d_3 \quad (4)$$

In the formula,  $\omega = [\omega_1 \ \omega_2 \ \omega_3]^T$  represents the angular velocity of the spacecraft relative to the inertial coordinate system in this system, and  $\Theta = [\varphi \ \theta \ \psi]^T$  represents the attitude angle vector of the spacecraft;  $I_i (i=1,2,3)$  represents the three principal moment of inertia of the spacecraft, and  $u = [u_1 \ u_2 \ u_3]^T$  represents the total controlling moment generated by the actuator and acting on the spacecraft body, and  $d = [d_1 \ d_2 \ d_3]^T$  is the external disturbing torque acting on the spacecraft. In particular, there are the following relationships in formula (1):

$$R(\Theta) = \begin{bmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \varphi & \sin \varphi \cos \theta \\ 0 & -\sin \varphi & \cos \varphi \cos \theta \end{bmatrix} \quad (5)$$

$$\omega_c(\Theta) = \omega_0 \begin{bmatrix} \cos \theta \sin \psi \\ \cos \varphi \cos \psi + \sin \varphi \sin \theta \sin \psi \\ -\sin \varphi \cos \psi + \cos \varphi \sin \theta \sin \psi \end{bmatrix} \quad (6)$$

When the spacecraft is in a small Euler angle motion, the kinematics formula (1) of attitude of spacecraft can be linearized as follows:

$$\dot{\omega}_1 = \dot{\phi} - \omega_0 \psi \quad (7)$$

$$\dot{\omega}_2 = \dot{\theta} - \omega_0 \quad (8)$$

$$\dot{\omega}_3 = \dot{\psi} - \omega_0 \phi \quad (9)$$

In the formula,  $\omega_0$  is the orbital angular velocity of the spacecraft.

In accordance with equations (7) - (9), the kinematics and dynamics formulas of attitude of spacecraft (1) ~ (4) can be rewritten as follows:

$$I \ddot{\Theta} + C \dot{\Theta} + K \Theta = u + d \quad (10)$$

In the formula,

$$I = \text{diag}(I_1, I_2, I_3)$$

$$K = \omega_0^2 \text{diag}(I_2 - I_3), 3(I_1 - I_3), I_2 - I_1)$$

$$C = \omega_0 (I_1 - I_2 + I_3) \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

The spacecraft taken in this paper is controlled by a four-inclined reaction wheel. Three reaction wheels are installed orthogonally to the axis of the spacecraft body, and a fourth flywheel is installed at an angle equal to the three axes of the system. In such a case, the mathematical model (10) of attitude of spacecraft can be finally rewritten into the following form:

$$I \ddot{\Theta} + C \dot{\Theta} + K \Theta = D\tau + d \quad (11)$$

In the formula, 1 is the actual output torque of the four reaction wheels, and matrix  $D \in R^{3 \times 4}$  is the control distribution matrix, which has the following relationship:

$$D = \begin{bmatrix} 1 & 0 & 0 & 1/\sqrt{3} \\ 0 & 1 & 0 & 1/\sqrt{3} \\ 0 & 0 & 1 & 1/\sqrt{3} \end{bmatrix}$$

However, the mathematical model of the attitude of spacecraft given by formula (11) doesn't take into account the problem of reaction wheel failure. As a matter of fact, the flywheel, as the main execution unit in the long-term orbit mode of the spacecraft, may produce four failure modes such as idling, stalling, increased friction torque and flywheel efficiency reduction.

### 3. Design of attitude of spacecraft fault-tolerant controller

#### 3.1 Failure reconstruction design of reaction wheels

In order to design the sliding mode observer to achieve the accurate reconstruction of the reaction wheel failure. Firstly, define the state variable  $x_1 = \Theta, x_2 = \dot{\Theta}$  and define the system output  $y = x_1$ , thereby, formula (13) can be rewritten as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0_{3 \times 3} & 1_{3 \times 3} \\ -I^{-1}K & -I^{-1}C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + I^{-1}u + I^{-1}f(t) \quad (14)$$

In the formula,  $0_{3 \times 3}$  is a  $3 \times 3$  zero matrix and  $1_{3 \times 3}$  a  $3 \times 3$  unit moment.

In such as case, in accordance with formula (14), the following theorem 1 can be drawn:

Theorem 1. For the attitude of spacecraft control system (14) of the reaction wheel failure, the sliding mode observer is designed as follows:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \lambda_1 \text{sgn}(y - \hat{x}_1) \\ \dot{\hat{x}}_2 = -I^{-1}K\hat{x}_1 - I^{-1}C\hat{x}_2 + \delta(\lambda_2 \text{sgn}(\tilde{x}_2 - \hat{x}_2)) + I^{-1}u \end{cases} \quad (15)$$

In the formula,  $\hat{x}_1$  and  $\hat{x}_2$  are the state estimates of state vectors  $x_1$  and  $x_2$ ,  $\tilde{x}_2 = \hat{x}_2 + (\lambda_1 \text{sgn}(x_1 - \hat{x}_1))_{eq}$ ; if  $x_1 - \hat{x}_1 \neq 0$ , then  $\sigma = 0$ , otherwise  $\sigma = 1$ . Then through choosing appropriate positive parameters  $\lambda_1$  and  $\lambda_2$ ,  $\hat{x}_i (i=1,2)$  will be converged to  $x_i$  in a limited time, and the reaction wheel fault  $f(t)$  will be reconstructed in a limited time accurately.

Proof. Define the system state estimation error  $e_1 = x_1 - \hat{x}_1$  and  $e_2 = x_2 - \hat{x}_2$ . In accordance with formulas (14) - (15), then the following formula can be drawn.

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} e_2 - \lambda_1 \text{sgn}(e_1) \\ -I^{-1}Ke_1 - I^{-1}Ce_2 - \delta(\lambda_2 \text{sgn}(\tilde{x}_2 - \hat{x}_2)) + I^{-1}f(t) \end{bmatrix} \quad (16)$$

In accordance with formula (16), the conclusion of theorem 1 is proved from the following two steps:

Step 1. Select Lyapunov function

$$V_1 = \frac{1}{2} e_1^T e_1 \quad (17)$$

From the first formula of formula (16), the derivative of formula (17) can be drawn:

$$\begin{aligned}\dot{V}_1 &= e_1^T \dot{e}_1 \\ &= e_1^T (e_2 - \lambda_1 \text{sgn}(e_1)) \leq -\lambda_1 (\|e_1\| - \|e_2\|)\end{aligned}\quad (18)$$

If the parameter  $\lambda_1$  is chosen,  $\lambda_1 \geq \|e_2\| + \varepsilon_1$  is valid. If the formula  $\varepsilon_1 > 0$  is workable, then

$$\dot{V}_1 \leq -\varepsilon_1 \|e_1\| \quad (19)$$

From formula (19), it can be seen that in a limited time:

$$T_1 \triangleq \frac{\|e_1(0)\|}{\varepsilon_1} \quad (20)$$

The system state error  $e_1$  will be converged to sliding mode surface  $e_1 = 0$ . When the sliding mode  $e_1 = \dot{e}_1 = 0$ , the equivalent output  $(\lambda_1 \text{sgn}(x_1 - \hat{x}_1))_{eq} = e_2$  can be drawn from the first formula of formula (16).

Note 1. For the convenience of analysis, this paper uses  $\|\bullet\|$  for the 2 norm of vectors.

Step 2. It can be seen from step 1 that the system will reach the sliding mode  $e_1 = \dot{e}_1 = 0$  in a limited time  $T_1$ , and then formula (16) is as follows:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} e_2 - \lambda_1 \text{sgn}(e_1) = 0 \\ -I^{-1}Ke_1 - I^{-1}Ce_2 - \lambda_2 \text{sgn}(e_2) + I^{-1}f(t) \end{bmatrix} \quad (21)$$

In such a case, select the new Laypunov function, then

$$V_2 = \frac{1}{2} e_1^T e_1 + \frac{1}{2} e_2^T e_2 \quad (22)$$

It can be seen from step 1 that after the time  $t \geq T_1$ , then  $e_1 \equiv 0$ . Thus, combining formula (21) and derivation of formula (22), the following formula can be drawn:

$$\begin{aligned}\dot{V}_2 &= e_1^T \dot{e}_1 + e_2^T \dot{e}_2 \\ &= e_2^T (-I^{-1}Ke_1 - I^{-1}Ce_2 - \lambda_2 \text{sgn}(e_2) + I^{-1}f(t)) \leq -\|e_2\| \left[ \lambda_2 - \left\| -e_2^T I^{-1}Ce_2 - e_2^T (I^{-1}K)e_1 + I^{-1}f(t) \right\| \right]\end{aligned}\quad (23)$$

In such a case, choose the parameter  $\lambda_2$  so that it satisfies the following inequality:

$$\lambda_2 \geq \left\| -e_2^T I^{-1}Ce_2 - e_2^T (I^{-1}K)e_1 + I^{-1}f(t) \right\| + \varepsilon_2 \quad (24)$$

In the formula,  $\varepsilon_2 > 0$  is a constant, then in accordance with formula (23), the following formula can be drawn:

$$\dot{V}_2 \leq -\varepsilon_2 \|e_2\| \quad (25)$$

In a similar way, from formula (25), the following formula can be seen that in a limited time:

$$T_2 = \frac{\|e_2(T_1)\|}{\varepsilon_2} + T_1 \quad (26)$$

The system state error  $e_2$  will be converged to the sliding mode surface  $e_2 = 0$ .

In accordance with the analysis results of step 1 and step 2, after a limited time  $T_2$ , all the observation errors  $e_1$  and  $e_2$  will reach the sliding mode surface  $e_1 = e_2 = 0$ , that is, the state variable observations  $\hat{x}_1$ 、 $\hat{x}_2$  can achieve accurate estimation of the attitude angle  $x_1$  and angular velocity  $x_2$  of the spacecraft. In particular, when  $\forall \geq T_2, e_1 = e_2 = 0$  and  $\dot{e}_1 = \dot{e}_2 = 0$  are valid, so in accordance with formula (16) the following formula can be drawn:

$$I^{-1}f(t) = (\lambda_2 \operatorname{sgn}(e_2))_{eq} \quad (27)$$

That is,

$$f(t) = I(\lambda_2 \operatorname{sgn}(e_2))_{eq} \quad (28)$$

In accordance with formula (28), the equivalent control term  $(\lambda_2 \operatorname{sgn}(e_2))_{eq}$  can be used to reconstruct the spacecraft reaction wheel fault value accurately as well as the external disturbing torque.

It can be known from the designed attitude controller that the attitude controller doesn't need to react the fault information of the flywheel reconstruction, so the correctness of the flywheel fault reconstruction information and delay between fault reconstruction and controller implementation won't have any influence on the attitude control algorithm put forward in this paper.

#### 4. Simulation results

In order to verify the effectiveness of the diagnosis of losing effectiveness and reconstruction method of the actuator designed in this paper, it is applied to a certain spacecraft for simulation analysis. The spacecraft's rotational inertia is  $I = \operatorname{diag}(12.49, 13.85, 15.75) \text{ kg} \cdot \text{m}^2$  and the external interference force rectangular formula given in [7] is adopted in this paper, that is:

$$\begin{aligned} d_1 &= A_0(3 \cos(\omega_0 t) + 1) \\ d_2 &= A_0(1.5 \sin(\omega_0 t) + 3 \cos(\omega_0 t)) \\ d_3 &= A_0(3 \sin(\omega_0 t) + 1) \end{aligned} \quad (29)$$

In the formula,  $A_0$  is the amplitude of the disturbing torque,  $A_0$  is taken as  $1.5 \times 10^{-5} \text{ N} \cdot \text{m}$ ; the orbital angular velocity of the spacecraft is  $\omega_0 = 1.078 \times 10^{-3} \text{ rad / s}$ . The maximum reaction torque of the reaction wheel control is  $0.1 \text{ N} \cdot \text{m}$ .

In the simulation, the initial parameters of the spacecraft are set as follows: the initial angular velocity is  $\omega(0) = [0.02 \ 0.02 \ 0.02]^\circ / \text{s}$ ; the initial attitude angle is  $\phi(0) = \theta(0) = \psi(0) = 0.2^\circ$ ; and the relevant parameters of the sliding mode observer (15) and controller (30) are selected as follows:

$$\lambda_1 = 15, \lambda_2 = 10 \text{ and } k = k_1 = k_g = 10^{-5}.$$

Adopting the above controlling parameters as well as initial conditions, the simulation is performed in the following two different situations:

Case 1: Under the normal conditions of the four reaction wheels of the spacecraft, the design controller (30) and the optimal control allocation of the motif are adopted;

Case 2: When the spacecraft's four-reaction wheel has not completely failed and stuck together, the design controller (30) and motif optimal control allocation are adopted.

#### 5. Conclusion

In order to solve the problem of external disturbance torque and actuator failure of spacecraft, this paper presents a fault reconstruction and attitude fault-tolerant control method for over actuated

spacecraft under reaction flywheel failure. This method combines the finite time theory with the design of sliding mode observer and control law to ensure the real-time on-line reconstruction of actuator fault information, realize the accurate reconstruction of four kinds of faults, such as idling, stalling, increasing friction torque and decreasing flywheel efficiency. At the same time, the problem of limited amplitude and speed of actuator output torque is considered, and the optimal basic sequence control allocation method is adopted to schedule redundancy, ensure the normal and stable operation of the system.

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